Parton saturation at strong coupling :

What have we learnt from RHIC and AdS/CFT ?

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Introduction: **Why strong coupling ?**

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  - it will not exceed 2 to 3 GeV at LHC (forward rapidities)
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- Deep inelastic scattering at HERA : unitarity corrections become important at low virtualities $Q^2 < 1$ GeV^2
  - are they to be attributed to saturation? or to confinement?
  - what is the interplay between these two phenomena ?

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- Outstanding open problems which might be related :
  - early thermalization, large elliptic flow, large jet quenching

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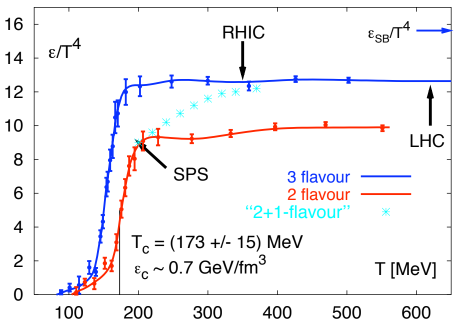
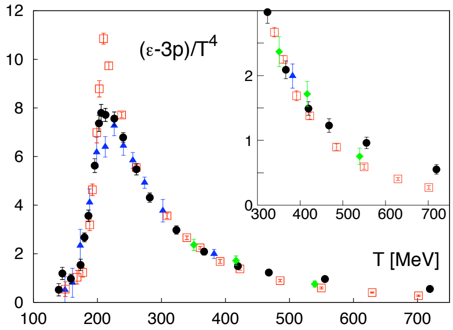
  - one cannot study saturation vs. confinement ☹
  - but one can study parton evolution/saturation in the quark–gluon plasma ! ☺

'Trace anomaly' in lattice QCD



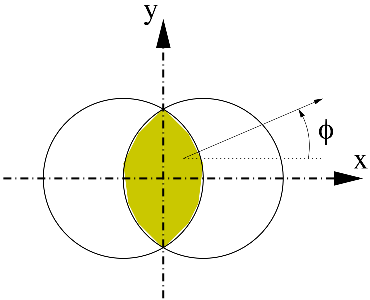


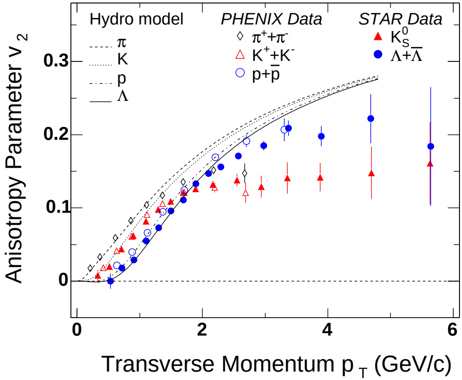
$$\beta(g)\,\frac{\mathrm{d}p}{\mathrm{d}g} = \langle T^{\mu}_{\mu}\rangle = \mathcal{E} - 3p$$

- $(\mathcal{E}-3p)/\mathcal{E}_0 \lesssim 10\%$ for any $T \gtrsim 2T_c \simeq 400$ MeV
- $\alpha_s \approx 0.3 \implies g \approx 2 \implies \lambda \equiv g^2 N_c \simeq 10$

Motivations: Heavy Ion Collisions @ RHIC & LHC

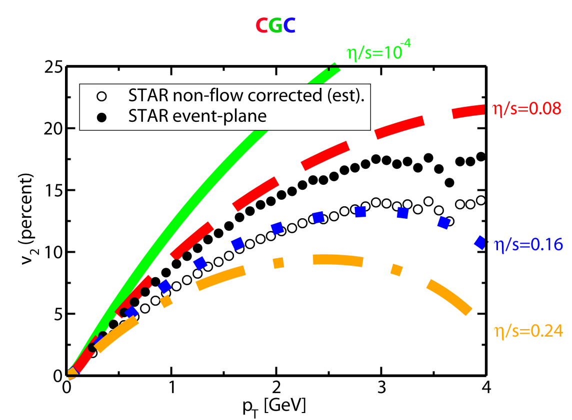
Elliptic flow





- Non–central AA collision: Particle distribution is not axially symmetric : $\mathrm{d}N/\mathrm{d}\phi \propto 1 + 2v_2 \cos 2\phi$ ($v_2 = 0$ for 'dust')
- RHIC finds a very large v_2 ! Natural for a liquid : Pressure gradient is larger along the smaller axis (x)

Hydro



(from Luzum and Romatschke, 2008)

- Hydrodynamical simulations can give estimates for v_2
- Too large viscosity kills v_2 !
- Good fits for very small viscosity/entropy ratio $\eta/s \sim 0.1$

Viscosity over entropy density ratio

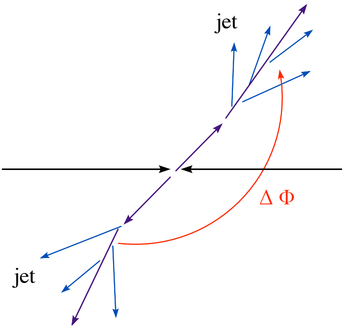
- Viscosity is smaller at strong coupling ! *(Maxwell, 1860)*
- Proportional to the mean free path $\ell \propto 1/\sigma \sim 1/g^4$
- Weakly interacting systems have $\eta/s \gg 1$ (in units of $\hbar$)
- AdS/CFT *(Policastro, Son, Starinets, 2001)*

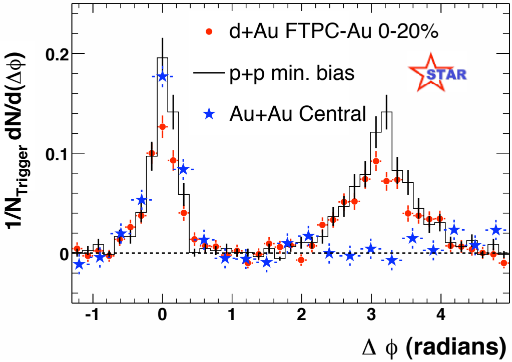
$$\frac{\eta}{s} \to \frac{1}{4\pi} \quad \text{when} \quad \lambda \equiv g^2 N_c \to \infty$$

- This is the limiting value allowed by the uncertainty principle
- The RHIC value is at most a few times $1/4\pi$!

  "strongly–coupled quark–gluon plasma" or *"perfect fluid"*

Jets in proton–proton collisions



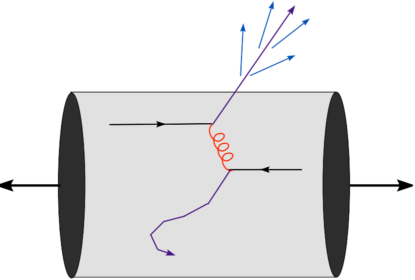


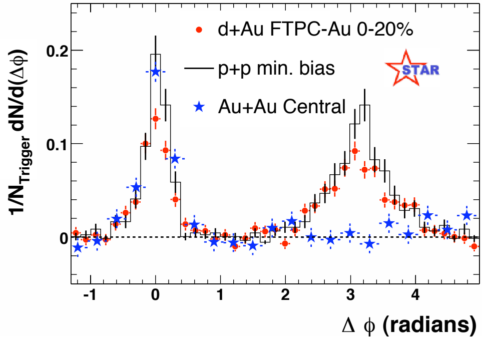
[Nucl.Phys.A783:249-260,2007]

- Azimuthal correlations between the produced jets:

  p+p or d+Au : a peak at $\Delta\Phi = 180°$

Jet 'quenching' in nucleus–nucleus collisions





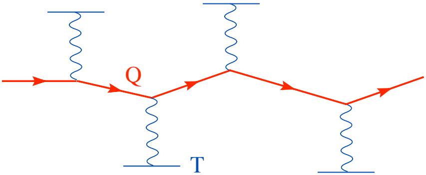
- The "away–side" jet has disappeared (for relatively hard transverse momenta: $Q \sim 2 \div 10$ GeV $\gg$ $T \simeq 400$ MeV)

  $\Longrightarrow$ strong interactions in the medium

- Perturbation theory seems unable to explain this suppression

Jet quenching in QCD at weak coupling

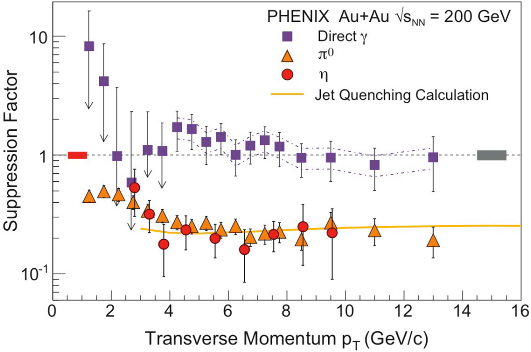
- Medium rescattering $\Longrightarrow$ transverse momentum broadening



$$\frac{\mathrm{d}\langle k_\perp^2\rangle}{\mathrm{d}t} \equiv \hat{q} \simeq \alpha_s N_c\, xg(x, Q^2)$$

- $xg(x, Q^2)$: gluon distribution per unit volume in the medium on the resolution scales $Q^2 \sim \langle k_\perp^2\rangle$ and $1/x \sim \Delta t_{\rm coh} T$
- If "medium" = QCD plasma at temperature T: we expect quarks and gluons with momenta $\sim T$
- Jet quenching requires parton evolution from T up to $Q \gg T$

How to measure $\hat{q}$?



Nuclear modification factor

$$R_{AA}(p_\perp) \equiv \frac{Yield(A+A)}{Yield(p+p)\times A^2}$$

- RHIC data seem to prefer a rather large value for $\hat{q}$:

  $\hat{q}_{\rm RHIC} \simeq 5 \div 15$ vs. $\hat{q}_{\rm pQCD} \simeq 0.5 \div 1\,{\rm GeV}^2/{\rm fm}$

  $\Longrightarrow$ 5 to 10 times larger than the pQCD estimate !

- … thus suggesting an enhanced parton evolution

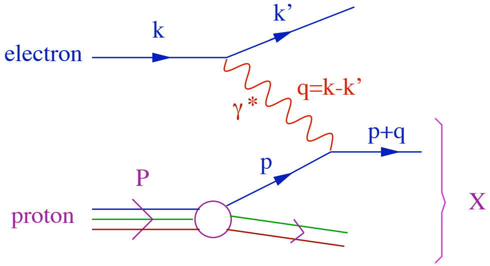
Deep inelastic scattering

- How to study parton evolution at strong coupling ?

- Compute DIS !
- 2 independent variables:

$$Q^2 \equiv q^\mu q_\mu \geq 0$$

$$x \equiv \frac{Q^2}{2P \cdot q}$$



- Structure function $F_2(x, Q^2)$: parton distributions for
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  - and longitudinal momentum $p_z = xP$

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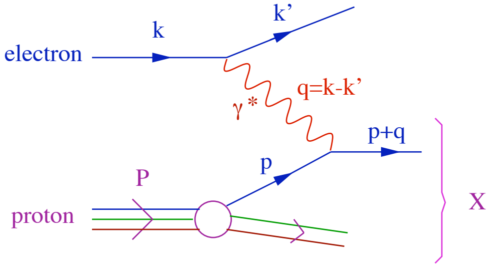


- Structure function $F_2(x, Q^2)$: parton distributions for
  - transverse size $\Delta x_\perp \sim 1/Q$
  - and longitudinal momentum $p_z = xP$
- How to compute DIS off a strongly coupled plasma ?

The AdS/CFT correspondance *(Maldacena, 1997)*

- A 'duality' (equivalence) between 2 very different theories
- A gauge theory ($\mathcal{N}=4$ SYM) in $D=3+1$ at strong coupling
  - SU(N_c), conformal invariance, fixed coupling, no confinement
- A string theory in $D=9+1$ $(AdS_5 \times S^5)$ at weak coupling
  - AdS_5 : Our physical world $(D=4)$ $\times$ a 'radial' dimension χ
- Strong 't Hooft coupling: $\lambda \equiv g^2 N_c \gg 1 \quad \& \quad g^2 \ll 1$
  - semiclassical limit of the string theory (gravity)
- $\mathcal{N}=4$ SYM at finite temperature $\Longrightarrow$ Black Hole in AdS_5
  - a Black Hole has entropy and thermal (Hawking) radiation

DIS off the Black Hole *(Hatta, E.I., Mueller, 07)*

- Virtual photon in 4D $\longleftrightarrow$ Maxwell wave A_μ in AdS$_5$ BH
- DIS cross section $\longleftrightarrow$ absorption of the wave by BH

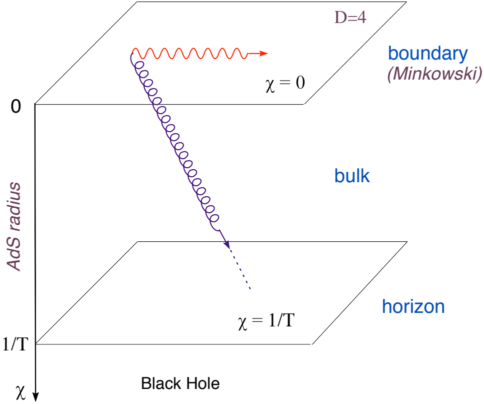
- Physical world: $\chi = 0$

  Black Hole horizon: $\chi = 1/T$
- Maxwell equations in AdS$_5$ BH

$$\partial_m\left(\sqrt{-g}g^{mn}g^{pq}F_{nq}\right) = 0$$

$$F_{mn} = \partial_m A_n - \partial_n A_m$$

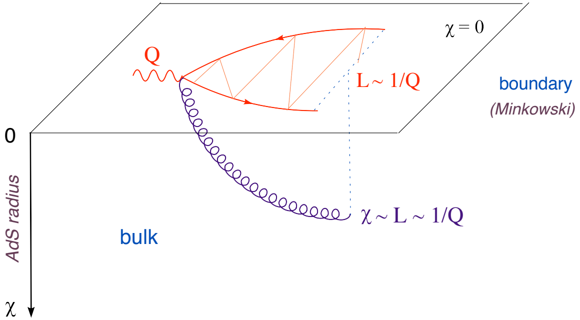
- No explicit coupling



The holographic principle

- What is the rôle of the 5th dimension ?

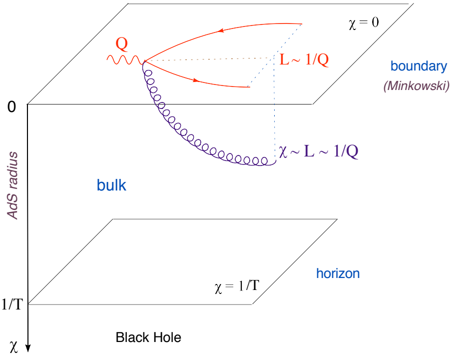
  Dual to the 'loop' momenta in the usual Feynman graphs

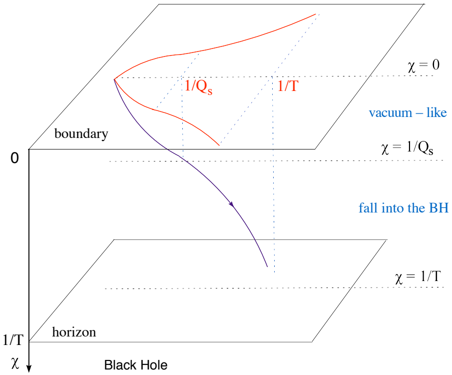


- Radial penetration χ of the space–like 'photon' in AdS$_5$ $\longleftrightarrow$ transverse size L of the partonic fluctuation on the boundary

Space–like photon in the plasma

- For low energies, the virtual photon does not ‘see’ the BH ...





- ... while for large enough energies, it is completely absorbed !

Saturation line

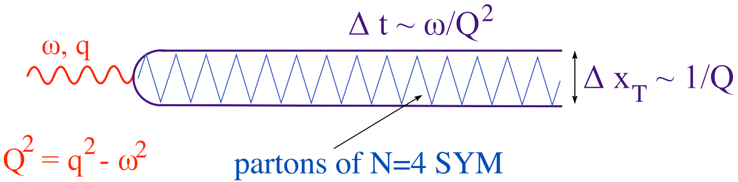
- Gravitational interactions are proportional to the energy density in the wave (ω) and in the plasma (T)

$$\text{DIS kinematics :} \quad x \equiv \frac{Q^2}{2\omega T} \quad \text{and} \quad Q \gg T$$

- Large ωT is tantamount to small Bjorken's x
- Critical ('saturation') value $x_s(Q) \simeq \frac{T}{Q} \ll 1$
  - $x > x_s \simeq T/Q$: $F_2(x, Q^2) \approx 0$: no partons !
  - $x < x_s \simeq T/Q$: $F_2(x, Q^2) \sim x\, N_c^2 Q^2$
- Consistent with the energy–momentum sum rule:

$$\int \mathrm{d}x\, F_2(x, Q^2) \simeq \Big[x F_2(x, Q^2) \Big]_{x=x_s} \sim N_c^2 T^2$$

Physical interpretation of Q_s



- Plasma acts on the (color–dipole) partonic fluctuation with a force $F \sim T^2$ which is pulling the system apart
- Partons on–shell when mechanical work $(F \times \Delta t) \simeq$ virtuality

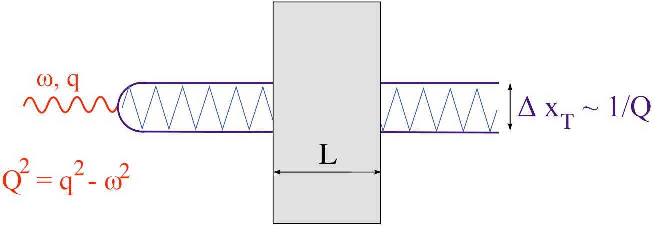
$$T^2 \times \frac{\omega}{Q^2} \sim Q \implies Q_s^2 \sim (\omega T^2)^{2/3} \sim \frac{T^2}{x^2}$$

- One factor of $1/x$ is just kinematics $(\Delta t \sim 1/xT)$

  The other one is graviton exchange : $1/x^{j-1}$ with $j = 2$

Saturation momentum for a 'nucleus'

(Mueller, Shoshi, and Xiao, 2008; Avsar, E.I., McLerran and Triantafyllopoulos, 2009; Kovchegov, 2010)



- Finite length "nucleus" (shock wave) with size $L \ll \omega/Q^2$

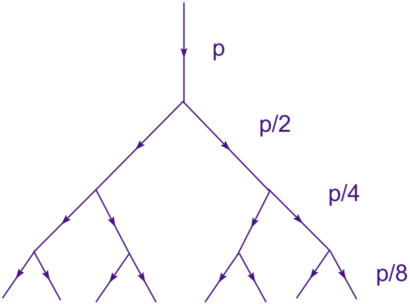
$$Q_s^2 \sim \frac{L\Lambda^3}{x}$$

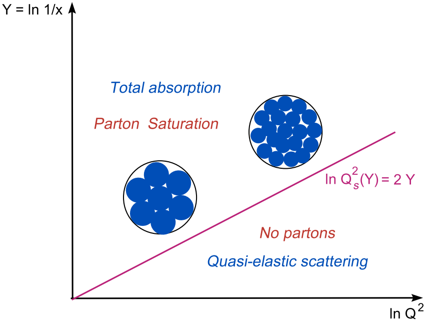
- ... to be compared to $Q_s^2 \sim 1/x^{0.3}$ from fits to HERA data

  'BFKL Pomeron' : gluon ladder (gluon spin is $j = 1$)

Parton evolution at strong coupling

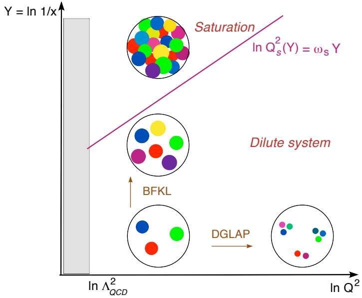
- All partons branch down to the smallest value of x consistent with energy conservation $\Longrightarrow$ no pointlike constituents

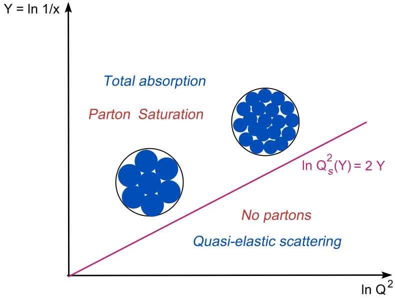




- The energy of the plasma is carried mostly by the partons along the saturation line: $x_s \simeq T/Q \ll 1$

Parton saturation: weak vs. strong coupling

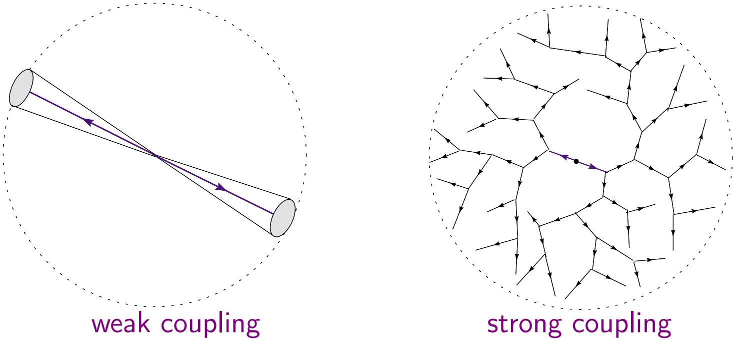




- Weak coupling : $Q_s^2(x) \propto 1/x^{0.3}$
  - $Q > Q_s(x)$: 'leading–twist' pdf
  - $Q < Q_s(x)$: $n \sim 1/\alpha_s$ (CGC)
- Strong coupling : $Q_s^2(x) \propto 1/x$
  - $Q > Q_s(x)$: no partons
  - $Q < Q_s(x)$: $n \sim 1$

No jets at strong coupling !

- No jets in e^+e^- annihilation at strong coupling !

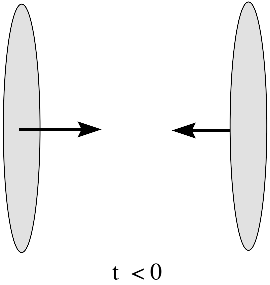


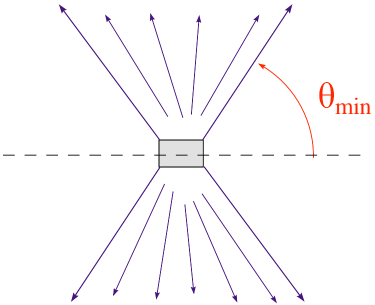
- An isotropic distribution of soft hadrons in the detector

  (similar conclusions by Hofman and Maldacena, 2008)
- Final state looks very different as compared to pQCD !

No forward/backward jets !

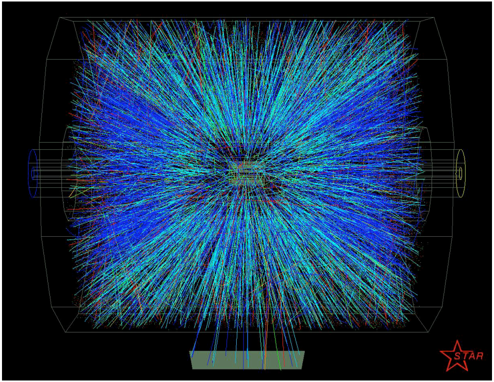
- No large–x partons $\Longrightarrow$ no hard ($Q \gg \Lambda$) particle production at forward/backward rapidities





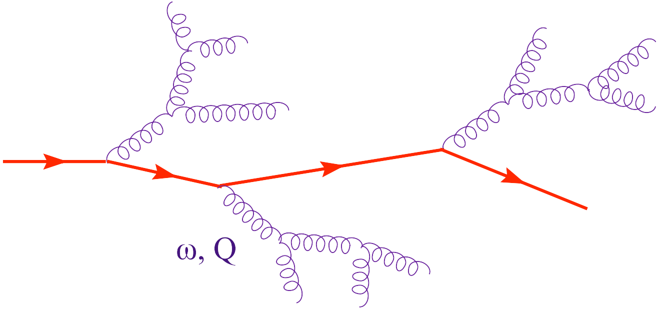
- All the energy is carried out by soft particles with $p \sim \Lambda$

Partons at RHIC



- Partons are actually 'seen' (liberated) in the high energy hadron–hadron collisions
  - central rapidity: small–x partons
  - forward/backward rapidities: large–x partons

Heavy Quark in a strongly–coupled plasma



- Medium–induced radiation
  - virtual quanta with $Q \lesssim Q_s$ are liberated into the plasma
  - energy loss, momentum broadening

$$-\frac{\mathrm{d}E}{\mathrm{d}t} \simeq \sqrt{\lambda}\,\frac{\omega}{(\omega/Q_s^2)} \simeq \sqrt{\lambda}\,Q_s^2 \simeq \sqrt{\lambda}\,\gamma\,T^2$$

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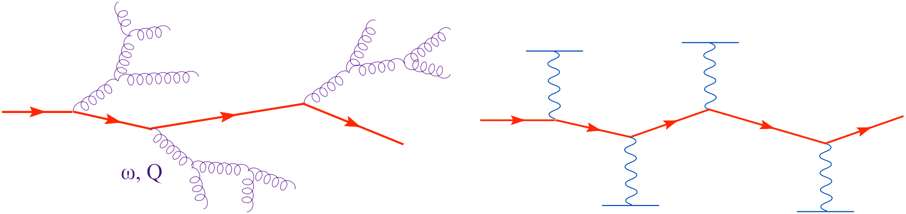
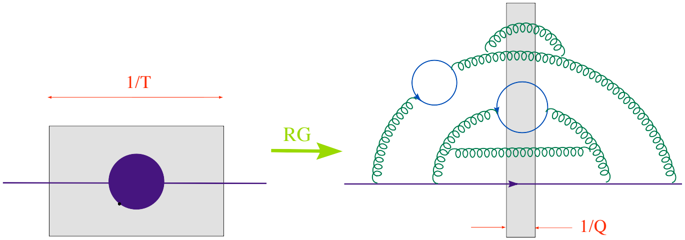


- Different mechanism than in pQCD: radiation vs. rescattering

  Casalderrey-Solana and Teaney, 2006; Gubser, 2006; Dominguez, Marquet, A. Mueller, B. Wu, B.-W. Xiao, 2008; G. Giecold, E.I., and A. Mueller, 2009 ...

Why gravity ? *(Polchinski and Strassler, 2001)*

- Why should gravity describe gauge theory at strong coupling ?
- OPE for DIS: Partons $\longleftrightarrow$ 'twist–2' operators
- The operators depend upon the resolution scale



- $\lambda \rightarrow \infty$: rapid evolution $\Rightarrow$ all operators are suppressed
- ... with one exception: the energy momentum tensor $T^{\mu\nu}$

$\Longrightarrow$ the effective theory for scattering must be gravity !

AdS/CFT applications to heavy ion collisions

- A very active area of research, with many interesting results
  - new perspectives on old problems: QGP = Black Hole
  - long–range properties (hydro, thermalization, etc) are likely controlled by stronger coupling
  - parton saturation is a universal phenomenon
- ... and some serious limitations :
  - no confinement, no asymptotic freedom
  - so far, no systematic way to improve (finite N_c)
  - high-energy physics looks very different from real–life QCD
- It teaches us the unity of physics
  - quantum field theory, nuclear physics, statistical physics, gravity, hydrodynamics ...